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Revised
December 10, 1964

NATIONAL BUREAU OF STANDARDS
Pressure Measurements Section

SUPPLEMENT FOR REPORTS ON DEAD WEIGHT PISTON GAGES

1.0 REDUCTION OF OBSERVATIONS ON DEAD WEIGHT PISTON GAGES

Measurements of pressures to an accuracy of a part in 10,000 or better can be made with a dead weight piston gage in good working order. To do so one must take into account a number of parameters of the instrument and its environment. The principal instrumental parameters are determined in the calibration. The user must determine those of the environment.

1.1 Pressure Equation. The pressure developed by a dead weight piston gage at its reference level is given by the formula:

$$p_p = \frac{\frac{M_m}{A_o}\left(1 - \frac{\rho_a}{\rho_m}\right) kg_L + \frac{V(\rho_{fa} - \rho_a)}{A_o} kg_L + \frac{\gamma C}{A_o}}{[1+a(t-t_s)]\,(1+bp_p)\,[1+d(p_{zo} + S_z p_p - p_j)]} \qquad (1)$$

where,

p_p Pressure at the reference level in pounds per square inch,

M_m Mass of the weights, including the piston assembly, in lbs,

A_o Effective area (mean area of the piston and cylinder) in square inches, at atmospheric pressure, temperature t_s, and jacket pressure p_{zo},

ρ_a Mean density of the air displaced by the load in lbs per cu. in.,

ρ_m Density of the weights in lbs per cubic inch,

$k = \frac{1}{980.665}$,

g_L Local acceleration due to gravity in cm/sec^2,

V Volume of oil in cubic inches contributing to the load on the piston, (see Footnote 1).

ρ_{fa} Density of the pressure fluid at atmospheric pressure, in lbs per cubic inch,

γ Surface tension of the pressure fluid in lbs force per inch,

C Circumference of the piston assembly in inches at the surface of the pressure fluid,

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Footnote 1. The quantity, V, used herein, is equal to the quantity $A_e y_{fa} - V_{fa}$ used in Monograph #65.

Therefore, $V\rho_{fa} = (A_e y_{fa} - V_{fa})\rho_{fa} = M_{fa}$ (2)

a Fractional change in area per degree C, and taken as equal to the sum of the thermal coefficients of linear expansion of the piston and cylinder,

t Temperature of the piston gage in degrees C,

t_s Reference temperature at which the value for A_o is known,

b Fractional change in effective area for 1 psi change in pressure,

d Fractional change in effective area for 1 psi change in jacket pressure,

p_{zo} Jacket pressure in psi required to reduce the piston-cylinder clearance to zero when $p_p = 0$, (see Footnote 2)

S_z Rate of change of zero clearance jacket pressure, p_z, with measured pressure, p_p, in psi, (see Footnote 2),

p_j Jacket pressure in psi.

1.2 Load Correction. A correction in pounds to be added to the mass load on the piston may be required because of constant load errors of undetermined origin. Possible causes of these errors are air currents around the weights, helical scratches on the piston or cylinder or guide bearing (cork screwing), or eccentric load effect.

1.3 Fluid Head. The reference level of the piston gage is the level at which the pressure, p_p, is determined from equation (1). It usually happens that the gage being tested, or the point at which the pressure is to be measured, is not at the reference level of the piston gage. Correction should, therefore, be made for the pressure difference due to the head of fluid between these points.

When a liquid is used to transmit the pressure and h_{fp} is kept small, the pressure head, H, may be determined by the approximation:

$$H\ (liq) = -\rho_{fp}\ h_{fp} \qquad (4)$$

Where ρ_{fp} is the mean density of the liquid and h_{fp} is the height of the liquid column measured from the piston gage reference level. The results are in pounds per square inch when ρ_{fp} is in pounds per cubic inch and h_{fp} is in inches.

Footnote 2. The quantity, p_z, is used in Monograph #65 but the quantities p_{zo} and S_z are not. The relation between them is expressed by the equation,

$$p_z = p_{zo} + S_z p_p. \qquad (3)$$

When air at about 23°C is used to transmit the pressure and h_{fp} is kept small, the pressure, p, at the test level may be determined from the approximation

$$p = p_p(1 - 2.9 \times 10^{-6}\ h_{fp}) \qquad (5)$$

for h_{fp} in inches, p and p_p in psi.

1.4 Mass and Buoyancy. The mass of the piston and loading weights may be determined either as true mass or as apparent mass determined by weighing in air, having a density of 0.000043 lbs/cu in. by comparison with standard brass weights (density 0.303 lbs/cu in.). The mass of the load on the piston should be reduced by an amount equal to the mass of the air displaced by the weights. This is accomplished by the factor $(1 - \frac{\rho_a}{\rho_m})$ in equation (1). When true mass values are used, the actual value of density of the weights should be used for ρ_m, but when apparent mass values are used, ρ_m should be assumed to be 0.303 lbs/cu in. The density of the air at room temperature and sea level pressure is about 0.000043 lbs/cu in., and the mass under these conditions will be reduced by about one part in 7,000.

The mass of any oil in a cavity in the piston, or the mass of oil displaced by an enlargement of a piston of otherwise uniform cross-section between the cylinder and the free oil surface, should be added to or substracted from the load on the piston. When the submerged cross-section is less than A_o, V will be positive, and when it is larger than A_o, V will be negative.

The buoyancy resulting from variations of the cross-section of the piston below the cylinder are taken into account by choosing a reference level above or below the lower end of the piston so that the resulting head of oil is equal to the buoyancy correction.

The height of the reference level, with respect to the lower end of the piston, is equal to the difference between the actual length of the piston below the cylinder and the length of a piston of uniform cross-section A_o, and of equal volume. When the actual cross-section is larger than A_o, the reference level is below the lower end of the piston and when it is smaller than A_o, it is above.

1.5 Gravity. In a piston gage loaded by dead weights, as with a liquid column manometer, the pressure is proportional to the local value of gravity. Readings on these instruments are reduced to standard gravity by introduction in equation (1) of the factor kg_L, where k is 1/980.665. The absolute value g_L of the gravity is given approximately by the formula:

$$g_L = 980.616 - 2.5862 \cos 2\phi + .0058 \cos^2 2\phi - .000094\ h \qquad (6)$$

where ϕ is the latitude and h is the altitude above sea level in feet.

Because of variations in the density of rocks below the surface, the value of gravity at any location may differ substantially from that of the formula. Deviations in excess of 0.100 cm/sec^2 (one part in 10,000) are observed in mountainous areas of the U. S. Observed values at over 1,000 locations are given in U. S. Coast and Geodetic Survey Publication No. 244 (price 75 cents at the Superintendent of Documents, U.S. Government Printing Office, Washington, D. C. 20234). These are in the International (Potsdam) System, and should be reduced by 0.012 cm/sec^2 to obtain the absolute values of gravity.

Estimates of the value of gravity at particular locations can be made by the Geodesy Division, U.S. Coast and Geodetic Survey, Washington, D. C. For greatest accuracy, gravity meter observations can be made at the laboratory by a survey team from the U.S. Coast and Geodetic Survey, the U.S. Geological Survey, or a private geophysical organization. (Efficient accomplishment of this work demands a lead time of several months).

1.6 Elastic Distortion. The distortion of the piston and cylinder under pressure depends greatly on the design and materials, and may either increase or decrease the effective area by as much as a part in ten thousand at a thousand pounds per square inch. The factor $(1 + bp_p)$ in equation (1) accounts for the change in area as the pressure increases.

1.7 Calibration. The direct measurement of the diameters of some pistons and most cylinders is difficult to do with adequate accuracy, and calculations of the pressure coefficient of area may be unreliable. It is, however, possible to balance a piston gage against another with high precision, and when the effective area and pressure coefficient of one are known, the effective area, pressure coefficient, and load correction of the other can be determined. The usual method used by the National Bureau of Standards to calibrate piston gages, is to weigh the load and then determine the values of A_o and b by the balancing method. The values of V, C, and height of the reference level are computed from measurements of the piston and cylinder assembly. A value of load correction is reported only when it is believed to be characteristic of the instrument. Whenever possible the calibration is done with the piston and weights rotating by their own inertia after bringing the piston to operating speed by spinning the weights manually. When this method cannot be used, the drive system, supplied with the instrument is used.

The drive error, corkscrew error, and eccentric load error are functions of speed of rotation or oscillation. To keep them small the speed should be only great enough to maintain lubrication.

The best value of pressure is the mean of equal numbers of observations taken in each direction of rotation.

The instrument level should be adjusted so that the axis of rotation of the piston is vertical. Whenever possible the level is observed by placing a bubble level on the piston assembly and the instrument is adjusted so that the indication is unchanged when the piston and bubble level are turned together to any position.

Controlled clearance piston gages are calibrated by an indirect (absolute) method. The value of A_o is determined by measurement of the piston diameter.

The zero clearance jacket pressure, p_z, is determined at several loads by observing the fall rate at each of several jacket pressures for each load. The volume of pressure fluid beneath the piston is kept as small as possible, all connections are made leak tight and the temperature is kept constant at temperature t_s. The cube root of the fall rates at each load are plotted versus jacket pressure and the straight line portions are extrapolated to zero fall rate to obtain values of p_z for each load. The values of p_z are plotted against piston gage pressure and a straight line extrapolation to zero pressure gives the value of p_{zo}. The value of p_{zo} can be verified when the piston is large and air lubricated by turning the piston with the fingers and increasing the jacket pressure until the piston is locked. The slope of the p_z line is the value of S_z.

The value of the rate of change of area with jacket pressure, d, is obtained by balancing against another piston gage and observing the change in load that is required to maintain the balance when the jacket pressure is changed. This experiment is done at a constant temperature.

The determination of d is made at several loads and provides data for the determination of A_o. The value thus obtained should agree with the value determined by direct measurement of the piston within the limitations of the instrument performance.

The pressure coefficient of area, b, for controlled clearance piston gages, is determined from the elastic properties of the piston and the theoretical relation

$$b = \frac{3\mu - 1}{y} \qquad (7)$$

where μ is Poisson's ratio for the piston, and y is Young's modulus for the piston.

2.0 FACTORS AFFECTING PERFORMANCE OF DEAD WEIGHT PISTON GAGES

The performance of dead weight loaded piston gages should not limit the accuracy of measurements made with the instrument. At low pressures the uncertainty of the value of the area A_o is usually the limitation, and at high pressures the uncertainty in the value of the pressure coefficient of area b, may be the limitation. The reproducibility of a piston gage in good working condition should be better than one part in twenty thousand in order to achieve the greatest possible accuracy.

2.1 Eccentric Load Error. Erratic behavior has sometimes been observed if the weights are stacked off center. There may be a fluctuation of pressure in synchronism with the rotation, or a change of the mean pressure which depends on the speed of rotation, but not on the direction. These troubles are worse for instruments in which the weights are stacked in a tall pile on top of the piston. The "eccentric load error" is the most common cause of poor performance of piston gages. Errors in the pressure exceeding one part in a thousand have been observed. The magnitude of eccentric load error has been observed to be a function of the speed, but not of the direction of rotation of the weights, and depends on the eccentricity and magnitude of the load, the alignment of the piston, cylinder, and guide bearing and the clearance between the piston and cylinder and in the guide bearing, and the leveling of the instrument.

Design of the weights are an important consideration in reducing eccentric load error. The diameter should be large, so that the height of the stack does not exceed the diameter. Individual weights should be balanced and should nest or index on the piston so that the load is balanced.

Some techniques for reducing eccentric load error are as follows:

1. Level the piston gage so that the piston rotates about a vertical axis. This is done by placing a bubble level on the piston and adjusting the instrument so that the piston and bubble level can be rotated together to any position without any change in indication.

2. Stack the weights so that they are centered on the axis of rotation.

3. Avoid excessive speed of rotation.

2.2 Corkscrewing. A helical scratch or tool mark on the piston, cylinder or guide bearing of a piston gage may result in an error known as "corkscrewing". This is a function of the speed and direction of rotation of the piston. It is usually negligible, but in severe cases may amount to as much as one part in a thousand. Corkscrew error can be reduced by avoiding excessive rotational speed. The user should not fall into the habit of making all observations with the piston rotating in one direction. About half of the observations should be made with the piston rotating in each direction. The observer will then be in a position to notice the corkscrew error if it appears. He can, if he wishes, average readings taken with the two directions of rotation.

2.3 Liquid Buoyancy. The buoyancy of the pressure transmitting liquid acting upon the piston assembly can be accounted for if it is constant and not too large. In some cases the secondary guide piston passes through a cavity that may be partially or entirely filled with liquid. As the piston moves up and down and the oil level lowers and rises, the effect of buoyancy may vary from zero to as much as 0.5 psi. Use of a piston gage having variable buoyancy necessitates a technique whereby the buoyancy can be made reproducible and known.

2.4 Drive Error. There are numerous ways by which the piston may be driven in a rotational or oscillatory manner. Nearly all methods may impart a vertical component of force to the piston. This vertical component will be proportional to the torque required to drive the piston and will be a function of load, eccentricity of load, speed, friction, and level. The resultant error may be negligible or may be very large. One test to determine the magnitude of drive error is to compare the results obtained with the drive in operation,with the results obtained when the piston and load are coasting free from the drive. A suspended, non-rotating load may oscillate abnormally when the piston is rotated or oscillated at a particular speed. Such speeds should be avoided.

2.5 Weights. The weights should be nonmagnetic, solid, and preferably of a hard, nonporous metal, such as brass or stainless steel. The surface finish should be smooth, preferably polished. Other considerations such as balance, diameter, and indexing are discussed above in connection with eccentric load effect.

2.6 Friction. Friction in a piston gage reduces the sensitivity and reproducibility of the instrument to a marked degree. Excessive friction results from eccentric loading, improper leveling, misalignment of the piston-cylinder-guide bearing assembly and either excessive or insufficient clearance between the piston and cylinder. Friction in the bearing between the piston and yoke of a suspended, nonrotating load may also be excessive.

When the weights are loaded on the piston and set into rotation, they should continue to rotate for several minutes. The simplest and most revealing criteria of performance is the coasting time of a freely spinning loaded piston.

2.7 Leak Rate. When the piston gage is connected to a leak tight system and the piston set into rotation, the piston will fall slowly as a result of the leak between it and the cylinder. The rate of fall will depend upon the clearance, length of crevice, diameter of the piston, pressure, viscosity of the fluid and concentricity of the piston and cylinder. Re-entrant cylinder piston gages should have a fall rate maximum at a pressure about one half of the range. Simple cylinder piston gages usually have fall rates which increase with pressure. However, at very high pressures most pressure fluids exhibit a very rapid increase in viscosity with pressure so that the fall rate may not increase as rapidly as expected.

2.8 Ageing Effects. A dimensional change of the piston or cylinder which affects the effective area significantly will be accompanied by a large change in the rate of leak of fluid between the piston and cylinder. The leak may be measured by observing the rate of fall of the piston when the gage is connected to a tight system of small volume.

2.9 Fluid Viscosity. The effective area of the piston is not affected by the viscosity of the pressure fluids, but the sensitivity, fall rate, and wear rate are affected by the viscosity. The best viscosity is one that will be high enough for a reasonable fall rate but not so high as to make the spin time too short and cause sluggish operation.

2.10 Leaks. Leaks other than that between the piston and cylinder, may result in pressure drop in connecting lines or in excessive fall rate either of which might result in significant measurement error.

2.11 Line Restriction. Long or small diameter connecting lines or other restrictions may result in significant pressure drop when leaks are present. Ordinarily small diameter lines are not objectionable when the piston gage is connected to a tight closed system. When two piston gages are to be balanced against each other a restriction between them may result in slow insensitive operation. This nearly always is true when oil is used to transmit the pressure. An air lubricated piston gage connected to a large volume system or to another air lubricated piston gage may oscillate between the piston stops. The oscillations can be reduced by introducing a restriction in the line to obtain sufficient damping of the system so that a balance can be achieved.